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Multi-Attribute Structural Optimization Based on Conjoint Analysis

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Over the last 30 years, there have been tremendous advances in multidisciplinary design optimization techniques to find optimum solutions. Advances have come in areas such as reducing computational cost, developing algorithms for efficient sensitivity analysis, using function approximations, etc. Most of these efforts assumed a single objective function (attribute) and a multitude of constraints. The focus of this paper involves the designer preferences in the multidisciplinary design optimization. The concept of modeling preferences among multi-attribute alternatives is prevalent in consumer product marketing. In this paper, we adopt conjoint analysis, a popular technique used in marketing to assess consumer preferences. This technique is used to obtain part-worths of the attributes, which provide insightful knowledge of the product under study, and which are further used to create new products in the market. This paper presents integration of conjoint analysis and multidisciplinary design optimization applications. A cantilever beam, a fixed plate, and a composite lightweight torpedo are used as examples.

I. Introduction

FORMULATING a structural optimization problem consists of making a mathematical model that describes the behavior of a physical system. After describing the design variables, objectives, and constraints, which are influential in the optimization process, we often confine ourselves to a single objective formulation and allow other performance measures or attributes to be constraints. In solving some structural optimization problems, performance measures such as component weight, fundamental frequency, stress concentration factor, etc., play equivalent importance and demand formulation of the problem as a multi-attribute design case. In the literature, there are various multi-objective algorithms [1–3] developed for the multidisciplinary design environment. They range from the weighted-sum method, the constraint method, and physical programming to goal programming. The idea of bringing designer preferences into engineering design has been shown in recent years [4–6].

Li and Azarm [7,8] present an approach to design a product considering designer preferences, customer preferences, and market competition. The product design process is divided into design alternative generation and design alternative evaluation. They use conjoint analysis to generate the product design model. Pareto designs are generated from a multi-objective optimization to evaluate the designs. This can be computationally expensive. See et al. [9,10] developed the hypothetical equivalents and inequivalents method to elicit the state preferences from a designer. This method generates the weights for each attribute considering the preferences. The weights are obtained by formulating an optimization problem. This method is useful in selecting one alternative among a set of alternatives. Multi-attribute theory is used to analyze multi-objective problems, but there are issues related to utility independence and assessment [5], and the normalization used in the method has shortcomings [6]. Discrete choice analysis was used to make demand models based on demand, price, cost, etc. [6], but a fictitious objective function is used. Michalek et al. [11,12] have developed the analytical target cascading technique to connect marketing and engineering disciplines where the big problem is divided into subproblems. In the optimization formulation, constant weighting factors are used. The authors suggest using a weighting update method if the current weights do not satisfy the user-specified tolerance. A survey of methods for solving multidisciplinary design optimization (MDO) problems is presented by Marler and Arora [13]. They divided the methods into three main categories: methods with a priori articulation of preferences, methods with a posteriori articulation of preferences, and methods with no articulation of preferences.

The aforementioned methods develop realistic ways of including designer preferences by making demand models; however, advancement is needed for structural optimization problems. Structural optimization may involve computationally expensive simulation. The designer may not have the luxury to generate the whole Pareto frontier to make decisions. For cases that have a higher number of attributes, it is not practical to generate the Pareto frontier. This paper optimizes multiple attributes by including preferences from the designer to obtain a preferred design. The part-worths generated based on the preferences now act as weights in the optimization. These weights change during the iterations depending on the level of the attribute.

Conjoint analysis, which is a popular marketing method for modeling consumer preferences, is used in conjunction with the optimization. First, conjoint analysis is performed and part-worths for all the attributes are obtained by incorporating designer preferences. The part-worths act as a front end while performing the optimization. The optimization is formulated as a traditional gradient-based method, but the objective function is now dependent on the part-worths obtained from conjoint analysis. In Sec. II, we provide a basic understanding of multi-attribute optimization and the major challenge faced in the MDO environment. In Sec. III, some of the methods used to solve an MDO problem are discussed with their strengths and weaknesses. Section IV is dedicated to conjoint analysis and describes the details of the algorithm. Section V, which is the main contribution of this paper, discusses combining the tools of marketing and engineering optimization. In Sec. VI, a few numerical examples are discussed showing the advantages of the method. Finally, Sec. VII contains summary remarks.

II. Problem Formulation

The problem of multi-objective optimization subject to multiple constraints can be posed as follows:

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Minimize

$$F = [f_1(X), f_2(X), \cdots, f_q(X)]$$

subject to the constraints

$$f_j(X) = \bar{f}_j \qquad j = q+1, \ldots, m_e$$
$$f_j(X) \geq \bar{f}_j \qquad j = m_e+1, \ldots, k$$

and the side bounds on the design variables are

$$X \geq X^l \tag{1}$$

where $f_i, i = 1, \ldots, q$ are competing objectives to be minimized, k is the total number of attributes, n is the number of design variables, and $\bar{f}$ are the limits of the constraints. Superscript l denotes the lower bound and X denotes the vector of design variables. According to the Pareto concept [14], in solving the preceding problem, a series of efficient or nondominated solutions can be obtained, with no unique optimal solution. These nondominated solutions form a set of solutions in which no decrease can be obtained in any of the attributes without simultaneously increasing at least one of the remaining attributes. The Pareto frontier is a line for two attributes, a surface for three attributes, and a $k-1$ dimensional surface for k attributes. Figure 1 shows the nondominated and dominated solutions for two attributes.

Multi-Attribute Optimization: As mentioned in the preceding section, we can generate a series of nondominated solutions. But in many applications, engineers are not interested in a series of solutions but are interested in one preferred solution. A higher level of information is needed to pick one solution from these set of solutions. Figure 2 shows a schematic diagram of multi-attribute optimization (MAO). MAO is a two-step process [15]. The first step optimizes multiple attributes, and a Pareto frontier with multiple tradeoffs is obtained. The second step is to select the best tradeoff solution.

In MDO applications, the designer does not have the luxury of having the whole Pareto frontier in front of him to make decisions. When the number of attributes are greater, there is a huge computational cost associated for generating the whole Pareto frontier. Also, in a few applications, such as simulation of a full crash, analysis takes around 24 h and, therefore, performing one optimization routine itself is computationally expensive. The designer will be picking the design based on the requirements and needs.

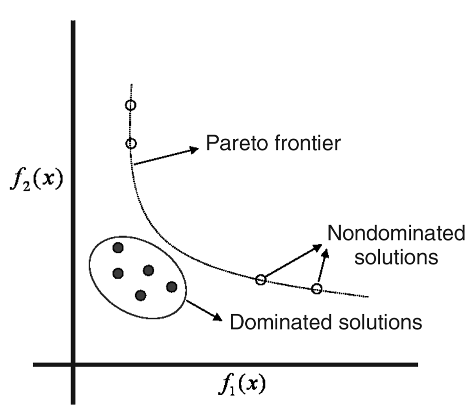

Fig. 1 Pareto frontier for two-attribute problem.

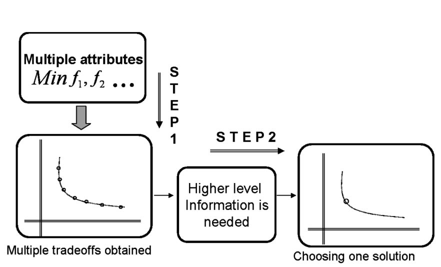

Fig. 2 Schematic representation of multi-attribute optimization.

III. Solution Procedures

In the following section, two of the commonly used MDO methods are discussed with their advantages and disadvantages.

A. Constraint Method

The constraint method is the most basic method. In this method, one of the attributes is converted into an objective function and the rest of the attributes are considered as constraints, resulting in the following formulation:

Minimize

$$f_1(x)$$

subject to

$$f_i(x) \leq f_i^{\max} \qquad i = 1, 2, \ldots, k-1 \tag{2}$$

The advantages of this method are easy setup and the availability of many optimization algorithms to solve the problem. But the disadvantage is that there are no tradeoffs involved in the optimization. For example, consider attributes f_1 and f_2 as cost and comfort level, respectively. The problem is formulated as minimize f_1 subject to $f_2^{\max} \leq 50\%$. We might end up with a design of \$100 cost with a comfort level of 50%, but this method will discard a design with a cost of \$105 for a comfort level of 90%. The latter design may be a preferred design. This is because there are no preferences set by the designer. The only input from the designer is on constraint limits. The iterative optimization is stopped when any of the constraints is active. This drawback is shown graphically in Fig. 3.

B. Weighted-Sum Method

The weighted-sum method is also popular in multi-objective optimization. In this method, each attribute is given a weight, and the objective function is formed by a weighted sum of the attributes. The objective is formed as

$$\sum_{i=1}^{k} w_i f_i$$

which is minimized subject to

$$\sum_i w_i = 1$$

These weights now represent the preferences of the designer. Weights are assigned based on the importance of each attribute. But the disadvantage is that these weights remain constant throughout the optimization iterations, which means constant tradeoffs. This might not be practical in real applications. To illustrate this, we use the same example of cost and comfort level discussed previously, where f_1 is

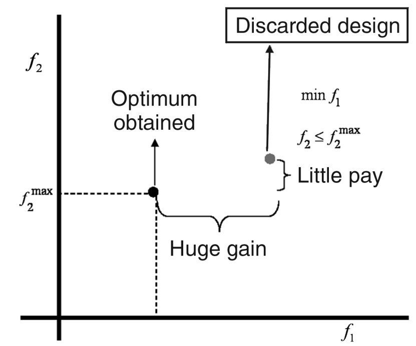

Fig. 3 Drawback of constraint method.

cost and f_2 is comfort level. The weights are selected as $w_1 = 0.75$ and $w_2 = 0.25$. The designer is willing to trade off $0.75/0.25 = 3$ units of cost for an increase of one unit of comfort level, regardless of the current value of the comfort level. Obviously this is not true, because it seems intuitive that anyone will tend to trade off less at a higher comfort level. Different weights can be used to obtain better optimum, but this would increase the computational cost tremendously.

IV. Conjoint Analysis Algorithm

The disadvantages of the current methods show the need for including designer preferences explicitly in MDO applications. In marketing science, the concept of modeling consumer preferences among multiple attributes has received much attention. The idea is to characterize a product into a bundle of attributes and assign levels for each attribute. A technique known as conjoint analysis is used to obtain the numerical value of the attributes of a product. Conjoint analysis can be defined as a technique that breaks down attributes to derive the part-worth associated with each level of a product based on the overall preferences a group of respondents assign to choice alternatives [16].

Conjoint analysis can be best understood by an example. The product under consideration is an automobile, and the attributes that are influencing customers to make judgments about buying the car are 1) make, 2) price, 3) seating capacity, etc. Yet many respondents find it very difficult to estimate the contribution of each attribute to the final decision. Conjoint analysis attempts to handle this problem by estimating the values of each attribute by calculating part-worths on the basis of preferences respondents make, along with product concepts that are varied in a systematic way. In the marketing discipline, conjoint analysis can be applied in health care, energy policies, and public policy decisions, to name a few areas. Further details are provided by Green and Srinivasan [17]. In engineering, conjoint analysis is applied for acoustic design by Grissom et al. [18].

This section provides details of the algorithm. Figure 4 shows the flowchart of the procedure. The first step in the process is to determine the attributes (or functions) f_i, where there are k number of attributes, that are relevant in the design. Next, for each of the attributes, levels must be chosen f_{ij}, where j represents the level number for ith attribute. The third major decision is to choose the number of combinations of attribute levels to include preferences. For example, one can choose a full factorial design that considers all the combinations. But if there are too many combinations, a fractional factorial design can be considered, as a full factorial design places a burden on the respondent for making preferences. The fourth step is to create a presentation form for the respondents to make their judgments. These forms can be in a pictorial format, a verbal format or a questionnaire format, etc. Also, the nature of the judgments from the respondent can be ratings on a 1–10 rating scale, or the judgments can be rankings, such as ranking all the products in increasing order of preference. The final step is to systematically convert the preferences input data into part-worths for all levels of each attribute. Several methods exist for this step [19], and the method's selection is dependent on the nature of judgments. The dummy-variable regression technique [20] is used in this work. This technique converts the preferences of the customer systematically using regression analysis.

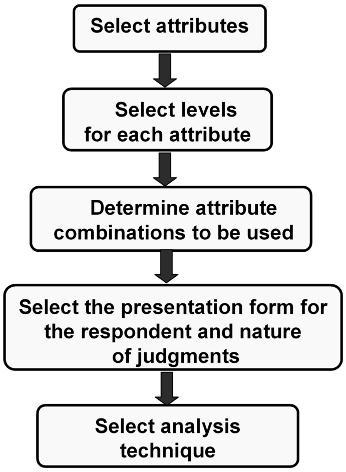

Fig. 4 Flowchart of conjoint analysis algorithm.

Algorithm Details: Each step of the algorithm is discussed in detail as follows. The marketing perspective is presented first, followed by an engineering viewpoint.

A. Selecting Attributes

In marketing, selecting attributes is a difficult task, because there are many attributes present, and it is important to know which ones to consider. For example, for an automobile, the attributes can be brand, color of the car, number of doors, car size, car power, fuel efficiency, tires used, vehicle warranty, engine life, cost, and so on. For engineering applications, this is not as much of a problem because when we design an aerospace structure, there are certain things we look for, such as mass, displacement, stress, flutter, natural frequency, etc., that become the attributes.

B. Determining Attribute Levels

In the aforementioned example of an automobile, and also in many marketing products, most of the levels are discrete. For example, the brand of the vehicle can be Ford, General Motors, or Toyota, and the number of doors can be two or four. Some of the attributes are continuous, like cost and engine life. For attributes having discrete levels, selecting the levels is trivial because we already know what the possible levels are. But for continuous attributes, selecting the levels is a difficult task. For example, cost of a product which has yet to be launched. It is continuous, but we make it discrete by choosing the levels which influence the customer. For engineering applications, selecting levels is also a difficult task because most of the attributes are continuous. Selecting the number of levels and the values for each requires an experienced designer. For example, we are designing a beam made up of aluminum. We know the stress has to be less than its yield strength, 30 ksi. Hence, one level for stress can be 30 ksi. The next question is how many levels should be considered and what should their values be. The designer needs to be experienced with the stress values range to select the number of levels and their values. A simple approach for making levels can be to select the range for stress and then linearly increase the stress from the lowest to the highest. This need for the designer to be experienced can be a drawback of this method.

C. Determining Attribute Combinations

The third step in the process of conjoint analysis is to decide the specific number of combinations of attributes that will be used to make preferences. According to Green and Srinivasan [17], at most five or six attributes can be used for a full factorial design. A higher number makes it difficult for the respondents to make their judgments, requiring the use of fractional factorial design. In this work, Sawtooth Software SMRT package is used to perform conjoint analysis. This package uses D-efficiency [21] for making the fractional factorial design. D-efficiency measures how close the design is relative to the hypothetical orthogonal design, and the design having the highest D-efficiency is chosen. If the design is orthogonal and balanced then it has a 100% D-efficiency. A design is orthogonal when the estimates are independent of the other terms in the model. A design is said to be balanced when each level occurs with equal frequency with each factor, which means the intercept is orthogonal to each effect. Imbalance increases the variance of the parameter estimates.

D. Selecting Respondent Presentation Form

This step involves selecting the form of presentation for the respondent to indicate his preferences. Three popular approaches are, verbal description, paragraph description such as questionnaires, and

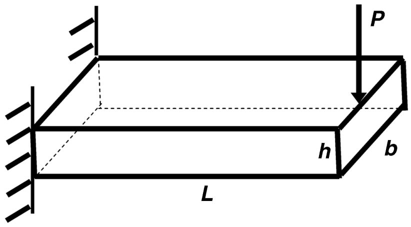

Fig. 5 Cantilever beam.

pictorial description. For engineering applications, it is very difficult to get descriptions of the attributes. Therefore, the designer has to make the tradeoff decisions based on the combinations of the attributes available.

The two most popular approaches for nature of judgment are ranking or rating. The ranking method ranks all the given combinations in increasing or decreasing order of preference. A rating scale of 1–10 can be used, judging each combination on this scale.

E. Dummy-Variable Regression Technique

The dummy-variable regression technique [20] is used to estimate part-worths and can be best understood by an example. Consider the cantilever beam shown in Fig. 5. Stress and mass are of importance, and so these are the two attributes. The levels for each attribute are chosen as mass → 0.2, 0.3, 0.4, and 0.5 lb, and stress → 27, 28, 29, and 30 ksi. The levels are chosen independently; one way to choose is that the designer knows that the stress has to be less than 30 ksi, and so, based on this, the other levels for stress are chosen. Stress levels are chosen with a constant increment because the designer has no real experience on how stress varies; therefore, constant interval is a safe bet. Also, lower stress values are chosen because they are preferred. The range of mass is expected based on the stress range, and the levels for mass are chosen appropriately. There are $4 \times 4 = 16$ combinations in all. Ranking scale preferences are used in this work, where rank 1 is least preferred and rank 16 is most preferred. The preferences are shown in Table 1. The first design of 0.2 lb and 27 ksi (and possibly other designs) is not practical, but is given a rank of 16 because this is the ideal design and, based on this, other ranking values are made. These rankings are designer preferences. The design of 0.2 lb mass and 28 ksi is ranked second best, and 0.3 lb mass and 27 ksi is ranked third best. This shows the designer is willing to have higher stress levels, because he is more concerned about mass. In this manner, the tradeoffs are incorporated in the rankings.

Table 1 Ranking preferences for cantilever beam

Mass, lb	Stress, ksi	Ranking	Mass, lb	Stress, ksi	Ranking
0.2	27	16	0.3	30	8
0.2	28	15	0.4	28	7
0.3	27	14	0.5	27	6
0.2	29	13	0.4	29	5
0.3	28	12	0.5	28	4
0.2	30	11	0.4	30	3
0.3	29	10	0.5	29	2
0.4	27	9	0.5	30	1

The next step is to convert these preferences in a systematic way into part-worths for all levels of each attribute. Sawtooth Software SMRT uses the dummy-variable approach to convert these data to fit into a model. In this method, for each ranking, if the level for an attribute is present, it is represented by one; otherwise, zero is substituted. The preceding rankings are now coded as shown in Table 2. The variables $x_1, x_2, \ldots, x_8$ are assigned for each level. The first four columns are for mass and the next four are for stress. Consider the first row of theTable 2: the data of rank 16 preference are converted by placing a one in the column of 0.2 lb mass and 27 ksi stress, while the rest of columns are substituted with zeros, because for this rank the levels that are present are only 0.2 lb mass and 27 ksi. Similarly, other ranking data are now converted systematically. Until now, the coding has been straightforward, but there is one complication that must be resolved: in regression analysis, no independent variable may be perfectly predictable based on the state of any other independent variable or combinations of independent variables, because the regression analysis procedure cannot separate the effects of confounded variables. There is a situation of linear dependency: knowing data about three levels of any attribute gives the information about the fourth level. To resolve this linear dependency, one column from each attribute is omitted. Any column from each attribute can be omitted. In this case, the first columns from both attributes are omitted, encompassing variables x_1 and x_5. The model to fit the data is selected as $y_L = \beta_0 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_6 x_6 + \beta_7 x_7 + \beta_8 x_8$, where β_2, β_3, and β_4 are the part-worths that need to be estimated, indicating the effect of each attribute level of mass on the overall preference. Similarly, β_6, β_7, and β_8 are the part-worths indicating the effect of each attribute level of stress on the overall preference. By omitting the columns of x_1 and x_5, we are assuming the corresponding βs as zero. In the model, β_0 is an intercept term, and y_L is the logit recoding [20] of the dependent variable, ranking. This recoding is a transformation of the rankings. The equation to convert ranking is

$$\text{Logit recode} = y_L = \ell n \frac{p}{1 - p} \tag{3}$$

where p is defined as probability, which is obtained as

$$p = \frac{y - \min + 1}{\max - \min + 2}$$

Table 2 Dummy-variable approach

$0.2(x_1)$	$0.3(x_2)$	$0.4(x_3)$	$0.5(x_4)$	$27(x_5)$	$28(x_6)$	$29(x_7)$	$30(x_8)$	Rank, y
1	0	0	0	1	0	0	0	16
1	0	0	0	0	1	0	0	15
0	1	0	0	1	0	0	0	14
1	0	0	0	0	0	1	0	13
0	1	0	0	0	1	0	0	12
1	0	0	0	0	0	0	1	11
0	1	0	0	0	0	1	0	10
0	0	1	0	1	0	0	0	9
0	1	0	0	0	0	0	1	8
0	0	1	0	0	1	0	0	7
0	0	0	1	1	0	0	0	6
0	0	1	0	0	0	1	0	5
0	0	0	1	0	1	0	0	4
0	0	1	0	0	0	0	1	3
0	0	0	1	0	0	1	0	2
0	0	0	1	0	0	0	1	1

in which y is ranking, and min and max are the minimum and maximum rankings, respectively. Other transformations available in Sawtooth Software's SMRT include no transformation or zero-centered transformation (subtract the mean). The rankings after transformation are shown in Table 3. The coefficients βs are solved using regression analysis. These βs represent the part-worths. The part-worths obtained after regression analysis are $\beta_0 = 2.59$, $\beta_2 = -0.98$, $\beta_3 = -2.31$, $\beta_4 = -3.29$, $\beta_6 = -0.62$, $\beta_7 = -1.29$, $\beta_8 = -1.91$. The intercept term β_0 is divided by the number of attributes $\beta_0/2 = 1.295$, and the quotient = 1 is added to each β, including those which were previously assumed to be zero (β_1, β_5). Hence, now the betas are $\beta_1 = 1$, $\beta_2 = 0.02$, $\beta_3 = -1.31$, $\beta_4 = -2.29$, $\beta_5 = 1.0$, $\beta_6 = 0.38$, $\beta_7 = -0.29$, $\beta_8 = -0.91$. These betas are now scaled. Sawtooth Software's SMRT uses zero-centered differences [20] for scaling. The details are as follows: β_1–β_4 represent the part-worth for mass and β_5–β_8 represent the part-worth for stress. The mean between β_1 and β_4 is calculated and subtracted from each β_1–β_4, as is similarly done for β_5–β_8. The part-worths obtained are shown in Table 4. This differences method rescales part-worths so that for each respondent, the total sum of the part-worths between the worst and best levels for each attribute is equal to the number of attributes times 100. This basically means that for each attribute, the difference between the best and worst levels are calculated; for mass it is 3.2861, for stress it is 1.9123, and the total sum of differences is 5.1985. Now, each zero-centered part-worth is scaled by 200/5.1985. Here, 200 is used because there are only two attributes. If there were three attributes, 300 would have been used. The final part-worths obtained are shown in Table 5. These part-worths now represent the preferences of the designer.

Table 3 Logit recode transformation of dependent variable, ranking

Ranking, y	Logit recode, Y_L
16	2.7726
15	2.0149
14	1.5404
13	1.1787
12	0.8755
11	0.6061
10	0.3567
9	0.1178
8	−0.1178
7	−0.3567
6	−0.6061
5	−0.8755
4	−1.1787
3	−1.5404
2	−2.0149
1	−2.7726

Table 4 Zero-centered part-worths before scaling

Mass, lb	Part-worth	Stress, ksi	Part-worth
0.2	1.6431	27	0.9562
0.3	0.6637	28	0.3388
0.4	−0.6637	29	−0.3388
0.5	−1.6431	30	−0.9562

Table 5 Part-worths for mass and stress

Mass, lb	Part-worth	Stress, ksi	Part-worth
0.2	63.21	27	36.79
0.3	25.53	28	13.03
0.4	−63.21	29	−13.03
0.5	−25.53	30	−36.79

V. Combining Marketing and Engineering Tools

The tools of marketing such as conjoint analysis are combined with optimization using engineering tools such as finite element analysis (FEA) or computational fluid dynamics (CFD). First, conjoint analysis is performed and part-worths are generated, and this acts as a front end to the optimization. There are three steps in this process which are discussed in the following.

A. Performing Conjoint Analysis

The first step is to perform conjoint analysis and generate part-worths. The preferences of the designer are included in this step. The outputs from this step are part-worths for each attribute at every level.

B. Making Linear Approximation

The part-worths obtained from the preceding step are discrete at selected levels. To perform optimization, part-worth values are needed for all combinations of attributes. For example, a structural finite element analysis computes attributes such as frequency, displacement, etc. For each of these attributes, corresponding part-worths need to be computed. Because part-worths are obtained at discrete level, piecewise linear interpolation and extrapolation are used to generate continuous part-worths with respect to each attribute. Linear approximation is not an exact prediction, but it is a fairly good approximation because, in engineering applications, several of the attributes have monotonic preferences. For example, lowering the stress is always better, and higher fundamental natural frequency is always good.

C. Formulating the Optimization Problem

The final step is to formulate the optimization. From the preceding step, different continuous part-worths are available. The optimization is formulated as follows:

Minimize

$$-\{p_1[f_1(X)] + p_2[f_2(X)] + \cdots p_k[f_k(X)] \qquad X^L \leq X \leq X^U \tag{4}$$

The actual problem is solved in terms of part-worths. For a given combination of design variables, there are attributes associated, and for every attribute value, there is a corresponding part-worth value, and the objective is the sum of these part-worths. Additive model of part-worths is assumed and is valid under certain assumptions [18]. A basic assumption is that the attributes are mutually preferentially independent (MPI); that means the tradeoffs between any pair of attributes f_i and f_j, keeping the levels of other attributes fixed, does not depend on fixed levels. For example, if there are three attributes, then the tradeoffs between any two are independent of the third. Details about the definition of MPI and existence of additive model are presented by Keeney and Raifa [22].

VI. Numerical Results and Discussion

Engineering applications, such as a cantilever beam, a fixed plate, and a composite lightweight torpedo, are used to demonstrate the method. A comparison with the constraint method is made for each example. MATLAB's optimization toolbox is used for performing optimization [23].

A. Cantilever Beam

The first example is the cantilever beam shown in Fig. 5. The beam is subjected to a tip load of $P = 100$ lb. Mass and tip displacement are the attributes considered. Breadth b and height h of the beam are the design variables. The closed form equations for mass and tip displacement are mass $= \rho bhl$, tip displacement $= d_{\text{tip}} = 4PL^3/Ebh^2$, respectively, where L is length of the beam = 20 in., ρ is density = 0.1 lb/in.^{3}, and E is Young's modulus = 10^7 psi. The constraint method optimization is formulated as follows:

Table 6 Levels for mass and displacement

No.	Mass, lb	No.	Displacement, in.
1	1.45	5	0.12
2	1.55	6	0.13
3	1.65	7	0.14
4	1.75	8	0.15

Minimize mass

$$d_{\text{tip}} \leq 0.15 \text{ in.} \qquad 0.5 \text{ in.} \leq \text{breadth, height} \leq 5 \text{ in.} \tag{5}$$

The optimum result is mass = 1.6219 lb and tip displacement = 0.15 in., with breadth = 0.5 in. and height = 1.6219 in..

Next, we solve the same problem using the conjoint-based approach. The levels chosen for mass and displacements are shown in Table 6 and are based on the designer's experience. The designer knows that the displacement has to be less than 0.15 in.; based on this, other levels for displacement are selected. Because the designer does not have enough experience to judge which levels for displacement are influential, the displacements are set to decrease linearly. For mass levels, the designer has an idea of what range of mass to use for the displacement levels, and so, based on this, other levels for mass are chosen. The other examples use similar reasoning to arrive at the levels used. Because there are only $4 \times 4 = 16$ combinations, a full factorial design is considered to make the rankings. Sawtooth Software's SMRT is used for the conjoint analysis, and the rankings are shown in Table 7. The higher the rank, the more the product is preferred. The rankings are chosen based on preferences. For a lower level of mass, higher rank is preferred, but for a higher level of mass, other attributes are preferred. Hence, from Table 7, the ideal combination is 1,5 and the second best is 1,6, which means the designer is willing to increase the displacement but not the mass, giving more importance to the mass for specified levels. The part-worths obtained after conjoint analysis are shown in Table 8. The part-worths obtained are discrete at specified levels. To obtain a part-worth for every attribute value, the preceding part-worths are linearly interpolated and extrapolated. The optimization is now formulated as follows:

Minimize

$$-(p_1 + p_2) \qquad 0.5 \text{ in.} \leq \text{breadth, height} \leq 5 \text{ in.} \tag{6}$$

where p_1 is the part-worth of mass, and p_2 is the part-worth of displacement. The optimum results are mass = 1.7963 lb and displacement = 0.1104 in., with breadth = 0.5 in. and height = 1.7963 in.. The optimizer has found the best tradeoff available considering the given preferences. The solution obtained is a preferred design. If different levels are set and rankings are changed, optimum results would be different. In the constraint method, the only preference from the designer is setting the upper limit on the constraints. There are no tradeoffs involved in the optimization. A comparison between both the results shows that for an increase of 10.75% in mass, a decrease of 26.4% in the displacement is achieved. This is obtained by a small increase in height of the beam with breadth remaining same. The rankings can be seen in Table 7. For lower levels of mass, more importance is given to a design with less mass, but at a higher level of mass, more preference is given to displacement. Hence, the optimizer had a better tradeoff for decreasing displacement than increase in mass.

Table 7 Preferences chosen for combinations of attribute levels

Rank	Combination	Rank	Combination
16	1,5	8	3,6
15	1,6	7	4,5
14	1,7	6	2,8
13	2,5	5	3,7
12	2,6	4	4,6
11	1,8	3	3,8
10	2,7	2	4,7
9	3,5	1	4,8

Table 8 Part-worths obtained for each level

Mass, lb	Part-worth	Displacement, in.	Part-worth
1.45	65.16	0.12	34.89
1.55	16.96	0.13	14.98
1.65	−22.70	0.14	−9.33
1.75	−59.42	0.15	−40.53

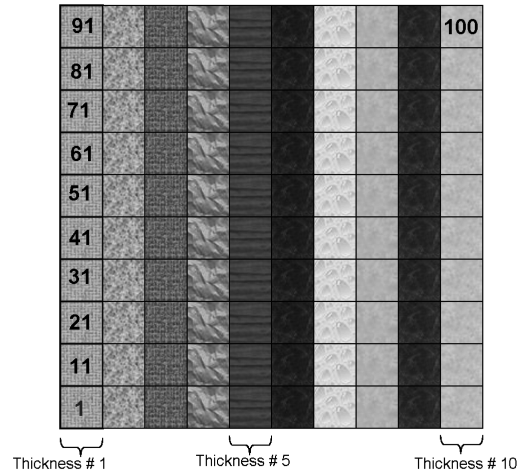

Fig. 6 Finite element model with element numbering.

B. Fixed Plate

The second example considered is a plate fixed at both ends with dimensions 10×10 in.. A uniformly distributed load of 100 psi is applied. The material properties used are Young's modulus $E = 10^7$ psi, density $\rho = 0.1$ lb/in.^{3}, and Poisson's ratio $\nu = 0.3$. The plate is modeled using quadrilateral plate elements; ABAQUS software is used. Because of double symmetry, a quarter model is modeled using 10×10 elements, and symmetry boundary conditions are placed. The finite element mesh and numbering are shown in Fig. 6. The design variables are thickness along the column of the plate, and this is shown with different shadings. There are 10 design variables. Mass, maximum displacement, and fundamental frequency are considered as attributes. A nonstructural mass of 0.5 lb is added to the quarter model structure, and distributed equally at each node. First, the optimization is solved using the constraint method by formulating the problem as follows:

Minimize mass

$$\text{displacement} \leq 0.05 \text{ in.} \qquad \text{fundamental frequency} \geq 8 \text{ Hz}$$
$$0.001 \leq X \leq 5 \text{ in.} \tag{7}$$

The optimum results are mass = 2.32689 lb, displacement = 0.00688 in., and frequency = 8.0024 Hz. Frequency is the driving constraint in the optimization, and the optimum design variables are shown in Table 9.

Solving the same problem using the conjoint approach, the levels for each attribute are shown in Table 10. The rankings for the previously chosen levels are shown in Table 11 and are based on the priorities of the designer. Lower mass level was preferred but, for a higher level of mass, other attributes dominate the preferences. The part-worths obtained for each level are shown in Table 12. These part-worths are linearly approximated as done earlier. The optimization problem is formulated as follows:

Table 9 Optimum thickness for fixed plate using constraint method

Thickness no.	Value, in.
1	0.7886
2	0.7668
3	0.8033
4	0.7752
5	0.8019
6	0.1756
7	0.6553
8	1.2281
9	0.6563
10	0.6563

Table 10 Attribute levels for mass, displacement, and frequency

No.	Mass, lb	No.	Displacement, in.	No.	Frequency, Hz
1	2.25	5	0.02	9	8
2	2.50	6	0.03	10	9
3	2.75	7	0.04	11	10
4	3.00	8	0.05	12	11

Table 11 Preferences for mass, displacement, and frequency attributes

Rank	Combination	Rank	Combination	Rank	Combination
30	1,6,12	20	2,8,11	10	2,6,9
29	2,5,12	19	1,8,10	9	4,7,11
28	1,5,11	18	3,6,11	8	2,7,9
27	2,5,11	17	3,7,11	7	1,8,9
26	1,7,12	16	2,8,10	6	4,5,10
25	3,5,12	15	3,6,10	5	4,8,12
24	2,7,12	14	1,7,9	4	4,8,11
23	1,6,11	13	4,6,12	3	4,6,10
22	1,5,10	12	3,5,9	2	4,5,9
21	3,8,12	11	3,7,10	1	3,8,9

Minimize

$$-(p_1 + p_2 + p_3) \qquad 0.001 \leq X \leq 5 \text{ in.} \tag{8}$$

where p_1, p_2, and p_3 are the part-worths of mass, displacement, and frequency, respectively. The optimum results are mass = 2.50005 lb, displacement = 0.003801 in., and frequency = 12.096 Hz. The optimum design variables are shown in Table 13. A comparison of the results from both methods is shown in Table 14. As seen before, the same trend of a little increase in mass resulting in a considerable decrease in displacement and increase in fundamental frequency is observed. This is achieved by a different distribution of thickness values when compared with the constraint method. In the constraint method, once one of the constraints became active, the optimizer obtained the lowest mass possible and it stopped the search. In the case of the conjoint approach, the optimizer searched the design space for the best possible tradeoff. The final optimum result is really dependent on the preferences of the designer. This approach has the designer more involved in the final result than the constraint approach, resulting in a preferred design. The rankings are given such that a design with a lower level of mass is preferred, but at a higher level of mass, more importance is given to the other two attributes. The optimizer ended up at a preferred design.

Table 13 Fixed plate optimum thickness using conjoint approach

Thickness no.	Value, in.
1	0.7548
2	1.9124
3	0.1521
4	0.1646
5	0.3124
6	0.0457
7	1.8125
8	0.6685
9	0.6386
10	1.5685

Table 14 Fixed plate results comparison

Attribute	Constraint method	Conjoint method	% difference
Mass, lb	2.3269	2.50005	7.44 ↑
Displacement, in.	0.0069	0.0038	44.78 ↓
Fundamental frequency, Hz	8.0024	12.096	51.15 ↑

C. Composite Lightweight Torpedo

1. *Modeling*

Torpedoes are self-propelled guided missiles that travel underwater and are designed to detonate on contact or in proximity to a target. A lightweight torpedo, similar to the MK-44 configuration of the U.S. Navy, was modeled using the finite element method. The overall length and diameter are 2.42 and 0.32 m, respectively. The finite element model was created using 1176 quadrilateral shell elements and 48 triangular shell elements. Figure 7 shows the finite element model. GENESIS, a finite element software, was used for modeling and analysis. To represent the total mass of the structural components inside the torpedo, concentrated mass elements were distributed along the nodes of the torpedo. Layered composite material properties are assigned to the outer shell. A conceptual design, modeled by Adduri et al. [24], with a sandwich honeycomb panel was used in this work. This model consists of the honeycomb surrounded by fiber-reinforced laminates on the top and bottom plates of the shell. Figure 8 shows the cross section of the shell. The stacking of the layers used are $0/\pm 45/90$ deg on both sides of the honeycomb. The material properties of the laminates are AS/3501 carbon/epoxy and are given in Table 15. The honeycomb can only handle transverse shear, hence only transverse shear moduli and transverse shear strength are defined for material properties, as shown in Eq. (9):

$$\begin{aligned} G_{13} &= 110 \text{ MPa}, \qquad G_{23} = 55 \text{ MPa} \\ S_{13} &= 0.65 \text{ MPa}, \qquad S_{23} = 0.40 \text{ MPa} \end{aligned} \tag{9}$$

2. *Attributes Considered*

Torpedoes travel underwater and should have enough strength to withstand pressure at crush depth. Hence, the composite shell should

Table 12 Part-worths for mass, displacement, and frequency

Mass, lb	Part-worth	Displacement, in.	Part-worth	Frequency, Hz	Part-worth
2.25	47.45	0.02	29.21	8	−65.72
2.50	33.68	0.03	5.68	9	−10.75
2.75	−4.14	0.04	−6.61	10	24.10
3.00	−76.99	0.05	−28.27	11	52.36

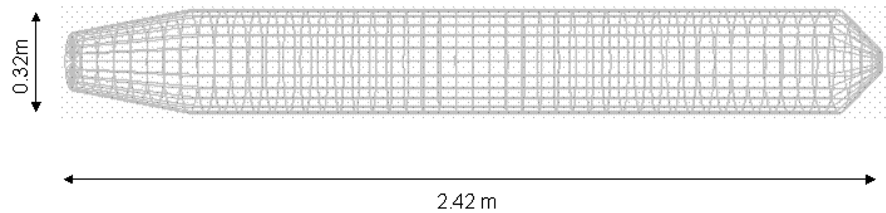

Fig. 7 Finite element mesh of composite lightweight torpedo.

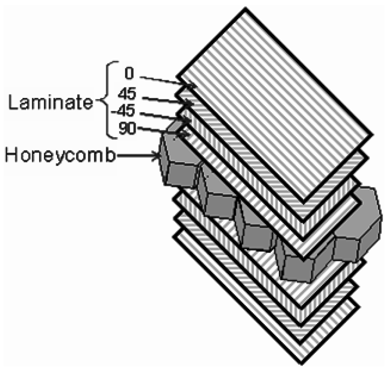

Fig. 8 Cross section of the composite shell.

be designed for crush depth. Because the considered torpedo is composite and not isotropic, the Holfman failure criterion is used. The failure index is defined as the ratio of applied stress to critical stress. Buckling is the second attribute. The critical buckling load factor is defined as the ratio of actual buckling load to the applied load. The fundamental natural frequency is the third attribute, and mass of the torpedo is the fourth attribute.

3. Optimization of Torpedo Hull

There are four design variables, which are the thicknesses of the laminates and the honeycomb. Symmetry is maintained on both sides of the honeycomb. The laminates on the top and bottom are linked. Also, ±45 deg layers take the same thickness. The design variables are thicknesses of the honeycomb, 0, ±45, and 90 deg layers. The constraint method is used and the optimization is formulated as follows:

Minimize mass

$$\omega_1 \geq 22.2 \text{ Hz} \qquad P_{\text{cr}} \geq 1.0 \qquad \text{FI} \leq 0.9$$
$$0.001 \leq X \leq 50 \text{ mm} \tag{10}$$

where ω_1 is the fundamental natural frequency, P_{cr} is the critical buckling load factor, and FI is the failure index of the critical layer. The results obtained are mass = 220.1303 kg, $\omega_1 = 22.2045$ Hz, $P_{\text{cr}} = 1.0002$, FI = 0.45. The optimum design variables are shown in Table 16. The buckling load factor and fundamental frequency are active constraints.

Now, using the conjoint approach, the attributes and levels for each are shown in Table 17. The products are ranked in order of preference as shown in Table 18. These rankings suggest that the higher level of mass is not preferred, and most importance is given to the mass attribute compared with all others. The part-worths obtained after conjoint analysis are shown in Table 19. The optimization problem for conjoint approach is formulated as follows:

Minimize

$$-(p_1 + p_2 + p_3 + p_4) \qquad 0.001 \leq X \leq 50 \text{ mm} \tag{11}$$

where p_1, p_2, p_3, and p_4 are part-worths for mass, ω_1, P_{cr}, and FI, respectively. The optimum results are mass = 226.0078 kg,

Table 15 Material properties of carbon/epoxy

Property	Carbon/epoxy
Longitudinal modulus, E_{11}	139 GPa
Transverse modulus, E_{22}	8.96 GPa
In-plane shear modulus, G_{12}	7.1 GPa
Poisson's ratio, ν_{12}	0.3
Laminate density, ρ	1600 kg/m^3
Longitudinal tensile strength, F_{1t}	1447 MPa
Longitudinal compressive strength, F_{1c}	1447 MPa
Transverse tensile strength, F_{2t}	51.6 MPa
Transverse compressive strength, F_{2c}	206 MPa
In-plane Shear Strength, F_6	93 MPa

Table 16 Optimum thickness using constraint approach

Design variable	Value, mm
Honeycomb	50
0 deg layer	1.4
±45 deg layer	0.6
90 deg layer	0.03

Table 17 Attributes and levels for composite torpedo

No.	Mass, kg	No.	P_{cr}	No.	ω_1, Hz	No.	FI
1	222	5	1.1	9	22.2	13	0.6
2	224	6	1.2	10	23	14	0.7
3	226	7	1.3	11	24	15	0.8
4	228	8	1.4	12	25	16	0.9

Table 18 Ranking-based preferences for composite torpedo

Rank	Combination	Rank	Combination	Rank	Combination	Rank	Combination
39	1,8,11,15	29	1,6,11,14	19	1,5,10,13	9	4,7,10,15
38	1,8,12,16	28	3,8,9,14	18	2,5,12,15	8	4,5,12,14
37	1,7,12,14	27	3,7,12,14	17	4,8,12,13	7	4,8,9,16
36	2,8,11,15	26	3,7,11,13	16	3,6,10,13	6	3,5,10,16
35	2,8,10,14	25	2,7,10,16	15	1,5,9,16	5	3,5,9,15
34	1,7,10,14	24	1,6,9,15	14	2,5,9,14	4	4,7,9,15
33	3,8,12,16	23	2,5,12,13	13	2,5,11,14	3	4,5,11,13
32	2,8,9,13	22	3,7,11,16	12	3,6,9,14	2	4,6,11,16
31	1,7,9,13	21	2,7,9,16	11	3,8,10,14	1	4,6,10,16
30	3,8,10,15	20	2,6,12,15	10	3,5,12,15	——	——

Table 19 Part-worths obtained for composite torpedo

Part-worth	Mass, kg	Part-worth	P_{cr}	Part-worth	ω_1, Hz	Part-worth	FI
78.94	222	−61.77	1.1	−28.07	22.2	9.38	0.6
28.50	224	−35.88	1.2	−7.59	23	8.25	0.7
−3.91	226	23.53	1.3	8.83	24	−0.26	0.8
−103.53	228	74.11	1.4	26.83	25	−17.37	0.9

Table 20 Optimum thickness of composite torpedo using conjoint approach

Design variable	Value, mm
Honeycomb	49.9
0 deg layer	1.32
±45 deg layer	1.04
90 deg layer	0.029

Table 21 Composite torpedo results comparison

Attribute	Constraint method	Conjoint method	% difference
Mass, kg	220.1303	226.0078	2.67 ↑
Buckling load factor, P_{cr}	1.0002	1.5309	53.06 ↑
Failure index, FI	0.45	0.00265	94.11 ↓
Fundamental frequency, Hz	22.2045	22.9699	3.45 ↑

$\omega_1 = 22.9698$ Hz, $P_{cr} = 1.5309$, FI $= 0.0265$, and the optimum design variables are shown in Table 20. A comparison of both methods is shown in Table 21, indicating that huge dividends can be obtained in the buckling load factor and failure index for a small tradeoff in mass.

From Table 21, it can be seen that for a mass increase of 2.67%, the critical buckling load factor, which was active in the previous constraint optimization, has met the design goal with a 53.06% increase in safety limit. Furthermore, the failure index has significantly decreased, whereas there is a small increase in the fundamental frequency. Comparing the optimum thickness for both cases, the ±45 deg layer thickness is increased from 0.6 mm for the constraint approach to 1.04 mm for the conjoint approach, and other thicknesses remained almost the same. This increase in the ±45 deg layer thickness provides the strength for the hull to withstand the buckling pressure and the stresses that develop inside the hull. The preferred rankings are for a safe design. Higher mass is not preferred, hence the optimizer did not find a better tradeoff for higher level of mass. It was able to find the best tradeoff for a small increase in the mass.

VII. Summary

In this paper, a novel technique for preference-based optimization was developed. This technique finds a solution in the design space that has the best tradeoffs available. The levels and rankings set by the designer drive the optimization, resulting in a preferred design. Also, the computing cost to perform the conjoint analysis is negligible. The only effort involved in performing conjoint analysis is to come up with levels for the attributes and rank them. Once the part-worths are determined, the effort is similar to solving a standard design optimization problem. Unlike a single objective function, with the assistance of conjoint analysis, preferences are incorporated in a multi-attribute problem. A comparison with the constraint method shows the advantages of the new technique.

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References

[1] Gunawan, S., and Azarm, S., "Multi-Objective Robust Optimization Using a Sensitivity Region Concept," *Structural and Multidisciplinary Optimization*, Vol. 29, No. 1, 2005, pp. 50–60. doi:10.1007/s00158-004-0450-8

[2] McAllister, C. D., Simpson, T. W., Hacker, K., Lewis, K., and Messac, A., "Integrating Linear Physical Programming Within Collaborative Optimization for Multiobjective Multidisciplinary Design Optimization," *Structural and Multidisciplinary Optimization*, Vol. 29, No. 3, 2005, pp. 178–189. doi:10.1007/s00158-004-0481-1

[3] Messac, A., "Physical Programming: Elective Optimization for Computational Design," *AIAA Journal*, Vol. 34, No. 1, 1996, pp. 149–158.

[4] Hazelrigg, G. A., "Framework for Decision-Based Engineering Design," *Journal of Mechanical Design*, Vol. 120, No. 4, 1998, pp. 653–658.

[5] Thurston, D. L., "Real and Misconceived Limitations to Decision Based Design with Utility Analysis ," *Journal of Mechanical Design*, Vol. 123, June 2001, pp. 176–182. doi:10.1115/1.1363610

[6] Wassenar, H. K., and Chen, W., "Approach to Decision Based Design with Discrete Choice Analysis for Demand Modeling," *Journal of Mechanical Design*, Vol. 125, Sept. 2003, pp. 490–497. doi:10.1115/1.1587156

[7] Li, H., and Azarm, S., "Product Design Selection Under Uncertainty and with Competitive Advantage," *Journal of Mechanical Design*, Vol. 122, Dec. 2000, pp. 411–418. doi:10.1115/1.1311788

[8] Li, H., and Azarm, S., "Approach for Product Line Design Under Uncertainty and Competition," *Journal of Mechanical Design*, Vol. 124, Sept. 2002, pp. 385–392. doi:10.1115/1.1485740

[9] See, T. K., Gurnani, A., and Lewis, K., "Multi Attribute Decision Making Using Hypothetical Equivalents and Inequivalents," *Journal of Mechanical Design*, Vol. 126, Nov. 2004, pp. 950–958. doi:10.1115/1.1814389

[10] See, T. K., and Lewis, K., "Multi-Attribute Decision Making Using Hypothetical and Inequivalents," *ASME Design Technical Conferences, Design Automation Conference*, American Society of Mechanical Engineers Paper No. DETC02/DAC-0230, 2002.

[11] Michalek, J. J., Ceryan, O., Papalambros, P. Y., and Korean, Y., "Balancing Marketing and Manufacturing Objectives in Product Line Design," *Journal of Mechanical Design*, Vol. 128, Nov. 2006, pp. 1196–1204. doi:10.1115/1.2336252

[12] Michalek, J. J., Feinberg, F. F., and Papalambros, P. Y., "Linking Marketing and Engineering Product Design Decisions via Analytical Target Cascading," *Journal of Product Innovation Management*, Vol. 22, No. 1, 2005, pp. 42–62. doi:10.1111/j.0737-6782.2005.00102.x

[13] Marler, R. T., and Arora, J. S., "Survey of Multi-Objective Optimization Methods in Engineering," *Structural and Multidisciplinary Optimization*, Vol. 26, No. 6, 2004, pp. 369–395. doi:10.1007/s00158-003-0368-6

[14] Pareto, V., *Manualedi Economica Politica*, Societa Editrice Libraria, Milan; translated into English as *Manual of Political Economy*, edited by A. S. Schwier, and A. N. Page, 1971, A. M. Kelley, New York, 1906.

[15] Deb, K., *Multi-Objective Optimization Using Evolutionary Algorithms*, Wiley, New York, 2001, Chap. 1.

[16] Reutterer, T., and Kotzab, H. W., "Use of Conjoint Analysis for Measuring Preferences in Supply Chain Design," *Industrial Marketing Management*, Vol. 29, No. 1, 2000, pp. 27–35.

[17] Green, P. E., and Srinivasan, V., "Conjoint Analysis in Consumer Research: Issues and Outlook," *Journal of Consumer Research*, Vol. 5, No. 2, 1978, pp. 103–123. doi:10.1086/208721

[18] Grissom, M. D., Belugundu, A. D., Rangaswamy, A., and Koopmann, G. H., "Conjoint-Analysis-Based Multiattribute Optimization: Application to Acoustic Design," *Structural and Multidisciplinary Optimization*, Vol. 31, No. 1, 2006, pp. 8–16. doi:10.1007/s00158-005-0522-4
[19] Green, P. E., and Srinivasan, V., "Conjoint Analysis in Marketing: New Developments with Implications for Research and Practice," *Journal of Marketing*, Vol. 54, No. 4, 1990, pp. 3–19.
[20] SMRT Application Help, Sawtooth Software, Sequim, WA, 2006.
[21] Kuhfeld, W. H., Tobais, R. D., and Garratt, M., "Efficient Experimental Design with Marketing Research Applications," *Journal of Marketing Research*, Vol. 31, No. 4, 1994, pp. 545–557. doi:10.2307/3151882
[22] Keeney, R. L., and Raiffa, H., *Decision with Multiple Objectives: Preferences and Value Tradeoffs*, Wiley, New York, 1976, Chap. 3.
[23] MATLAB, *Optimization Toolbox Users Guide*, 2006 ed., MathWorks, Natick, MA, 2006.
[24] Adduri, P., Penmetsa, R. C., and Grandhi, R. V., "Optimization of Lightweight Composite Torpedo Structure with System Reliability Constraint," *Structural and Multidisciplinary Optimization* (submitted for publication 2005).

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